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Generalized *-derivations in Prime*-rings

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Abstract— In this paper it is proved that a prime*-ring R admits a generalized reverse*-derivation F associated with non-zero*-derivation d, then either [d(x),z] = 0 or F is reverse*-centralizer. Next it is aimed to prove that a Prime*-Ring R admits a generalized*-left derivation F with associated *-left derivation d then either R is commutative or F is Right *-multiplier.

Index Terms— prime*-ring,Jordan *-derivation, Generalized*-derivation, ,Generalized reverse*-derivation.

1 INTRODUCTION

The study of Derivations in rings through initiated long back,but got impetus only after posner.E.C [1] who in 1957 established two very striking results on derivations in prime rings.The study of *-rings using generalized derivations has become an innovative research topic during the last few decades leading to many excellent results and questions.

Shakir Ali [2] have defined the notions of generalized*-derivations &generalized reverse*-derivations and proved some theorems involving these mapping.Here it is presented some results on prime*-rings admits generalized*-derivations.An additive mapping $x \to x^*$ on a ring R is called an involution if it satisfies following axioms 1) $(x+y)^*=y^*+x^*$ 2) $(xy)^* = y^*x^*$ 3) $(x^*)^* = x$ $\forall x, y \in R$. A prime*-ring is defined as $xa^*y = 0$ implies either $x=0$ or $y=0$. An additive mapping $d:R\to R$ is called a reverse *-derivation if $d(xy) = d(y)x^*+ yd(x)$ holds $\forall x,y\in R$. An additive mapping $F:R\to R$ is called a generalized reverse *-derivation if $F(xy) = F(y)x^*+ yd(x)$ holds $\forall x,y\in R$. An additive mapping $F:R\to R$ is called a generalized *-left derivation if $F(xy) = y^*F(x)+ xd(y)$ holds $\forall x,y\in R$. An additive mapping $F:R\to R$ is called a generalized jordan *- derivation associated with if $F(x^2) = F(x)x^* + xd(x)$ for all $x \in R$.

2 THEOREMS AND PROOFS

Theorem 1: Let R be a prime *-ring.If R admits a generalized reverse *-derivation F with an associated non-zero reverse *-derivation then either $[d(x),z] = 0$ or F is reverse *- centralizer.

Proof: We are given that F is a generalized reverse *-derivation with an associated non-zero reverse *-derivation with an associated non-zero reverse *-derivation d,

we have $F(xy) = F(y)x^*+yd(x)$. (1)

Replace x by xz in equation(1)

$F(xzy) = F(y)z^*x^*+y(d(z)x^*+zd(x)$.

On the other hand

$F(xzy) = F(x(zy)) = F(zy)x^*+zyd(x) = F(y)z^*x^*+yd(z)x^*+zyd(x)$ (2)

Substracting (2) from (1) we get

$[y,z]d(x) = 0$. (3)

Replace y by d(x)z we get

$[d(x)z,z]d(x) = 0$.

$=d(x)[z,z]d(x)+[d(x),z] zd(x) = 0$.

$= [d(x),z] zd(x) = 0$.

Since R is prime we get either $[d(x),z] = 0$ or $d(x) = 0$ $\forall x\in R$.

Case1:_ if d=0 then F is left *-revese centralizer or $[d(x),z] = 0$ $\forall x,z\in R$

Theorem2:_ Let R be a prime*-ring. If R admits a generalized *-left derivation associated with *-left derivation then either R is commutative or F is right *-multiplier.

Proof: By the definition of generalized*-left derivation $F(xy) = y^*F(x)+xd(y) \forall x,y\in R$

Replace y by yz then $F(xyz) = F(x(yz)) = (yz)^*F(x)+xd(yz)$.

$= z^*y^*F(x)+x(z^*d(y)+yd(z)$.

$= z^*y^*F(x)+ xz^*d(y)+xyd(z)$ (4)

On the other hand

$F(xyz) = F(xy(z)) = z^*F(xy) +xyd(z)$

$= z^*y^*F(x)+z^*xd(y)+ xyd(z)$ (5)

Substracting (5) from (4) we get

$xz^*d(y)-z^*xd(y) = 0= [x,z^*]d(y) = 0$.

Replace $z^*\to z$ we get $[x,z]d(y) = 0$. (6)

Now again replace x by xz in (6) we get

$[xz,z]d(y) = 0=x[z,z]+[x,z]zd(y) = 0$.

$[x,z] z d(y) = 0$.

Since R is prime either $[x,z] =0$ or $d(y) = 0$.

We conclude that either R is commutative (or) F is right *-multiplier.

Lemma 1. Let R be a 2-torsion free non-commutative prime *-ring and Let $F:R\to R$ is called a generalized jordan *- derivation which satisfies $f(h)h+hd(h)\in Z(R)$ then $[f(hg+gh),y] =[f(h)g+hd(g),y]+[f(g)h+gd(h),y]$.

Proof. For any $r\in R$

$F(h^2) = F(h)h+hd(h) \in Z(R)$ (1.1)

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$F((h+g)^2) = F(h+g)(h+g) + (h+g)d(h+g)$
$= F(h)+F(g)(h+g) + (h+g)(d(h)+d(g))$
$= F(h)h + F(h)g + F(g)h + F(g)g + h d(h) + hd(g) + gd(h) + gd(g)\ \forall h \in H(R)$ (1.2)

Multiplying (1.2) by y on left side we get

$y F((h+g)^2) = y(F(h)h + h d(h)) + y(F(h)g + hd(g)) + y(F(g)h + gd(h)) + y(F(g)g + (gd(g)) \in Z(R)\ \forall h \in H(R)$ (1.3)

Multiplying (1.2) by y on right side we get

$F((h+g)^2) y = (F(h)h + h d(h)) y + (F(h)g + hd(g)) y + (F(g)h + gd(h)) y + (F(g)g + (gd(g)) y \forall h \in H(R)$ (1.4)

Comparing (1.3) and (1.4) we get

$[F((h+g)^2), y] = [F(h^2), y] + [F(g^2), y] + [F(h)g + hd(g), y] +] + [F(g)h + g d(h), y]$

Using (1.1) we get

$[F(hg+gh), y] = [F(h)g + hd(g), y] +] + [F(g)h + g d(h), y]$

Lemma 2. Let R be a 2-torsion free non-commutative prime*-ring and $d: R \to R$ be Jordan *-derivation which satisfies $d(h)h+hd(h) \in Z(R)$ then $[d(hg+gh, y] = [d(h)g+d(g)h+hd(g)+gd(h), y]$

Proof. $d(h^2) = d(h)h+hd(h) \in Z(R) \quad \forall h \in H(R)$ (2.1)

Replace h by h+g in (2.1)

$d((h+g)^2) = d(h+g)(h+g) + (h+g)d(h+g)$
$= (d(h)+d(g))(h+g) + (h+g)(d(h)+d(g))$
$= d(h)h + d(h)g + d(g)h + d(g)g + h d(h) + hd(g) + gd(h) + gd(g) \in Z(R)$
$= d(h^2) + d(g^2) + d(h)g + d(g)h + hd(g) + gd(h) \in Z(R)$
$\Rightarrow [d(h^2) + d(g^2) + d(h)g + d(g)h + hd(g) + gd(h), y] = 0\ \forall h \in H(R)$
$= [d(h^2) + d(g^2), y] + [d(h)g + d(g)h, y] + [hd(g) + gd(h), y] = 0$
$[d(h^{2+}g^2 + hg + gh), y] = [d(h)g + d(g)h + hd(g) + gd(h), y]$ (by 2.1)
$[d(hg+gh), y] = [d(h)g + d(g)h + hd(g) + gd(h), y]$

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5 CONCLUSION

Hence a prime*-ring R admits a generalized reverse*-derivation F with associated non-zero reverse*-derivation d then either F is reverse*-centralizer or commutator of d(x) and z equal to zero.If F is generalize *-left derivation then either F is Right*-mulitiplier or R is commutative.